9/7/2016 "Life is a journey, not a destination." -Steven Tyler

HW: Test 1 on Tuesday 9/20

AIM: How do we find limits analytically?

Warm Up:

Finding Limits by Analytic Methods

Observing the graph of a function only can be misleading at times when finding the limit of a function. It is possible to find limits using algebraic techniques and limit theorems.

You will learn to analyze limits by the following methods:

Methods to Analyze Limits:

1. Direct substitution.
2. Principal Limit Theorem
3. Factor, cancellation technique. Then go back to step 1.
4. The conjugate method, rationalize the numerator. Then, go back to step 2.
5. Special trig limits of $\lim_{x\to 0}\frac{\sin x}{x}=1$ or $\lim_{x\to 0}\frac{1-\cos x}{x}=0$
6. L'Hopitâls Rule (presented in Unit 3)

Substitution Theorem

(fraction)

If f is a polynomial function or rational function then $\lim_{x \to c} f(x) = f(c)$ provided that if f is a rational function the value of the denominator does not equal 0.

plug in the x-value and get your answer

EX #1: Find each of the following limits analytically using direct substitution.

A. $\lim_{x \to 2} \left(3x^2 - 5x + 4\right) = 3(2)^2 - 5(2) + 4 = 6$

B. $\lim_{x \to 2} \dfrac{x^3 + 1}{x + 1} = \dfrac{2^3 + 1}{2 + 1} = \dfrac{9}{3} = 3$

C. $\lim_{x \to e} \dfrac{\ln x}{3x} = \dfrac{\ln e}{3e} = \dfrac{1}{3e}$

D. $\lim_{x\to 4} \sqrt[3]{x+4}$ $= \sqrt[3]{4+4} = \sqrt[3]{8} = 2$

$\pi = 180°$

E. $\lim_{\theta\to\frac{\pi}{6}} \sin 2\theta$ $= \lim_{\theta \to \frac{180}{6} = 30°} \sin 2\theta = \sin 2(30) = \sin 60 = \frac{\sqrt{3}}{2}$

Calculator

F. $\lim_{x\to 5} \log_3(x+4)$ $= \log_3(5+4) = \boxed{\log_3 9} = 2$

Finding Limits of Functions at Undefined Values

Consider the following cases and what happens when you try to evaluate limits by direct substitution.

EX #2: The Factoring or Cancellation Technique

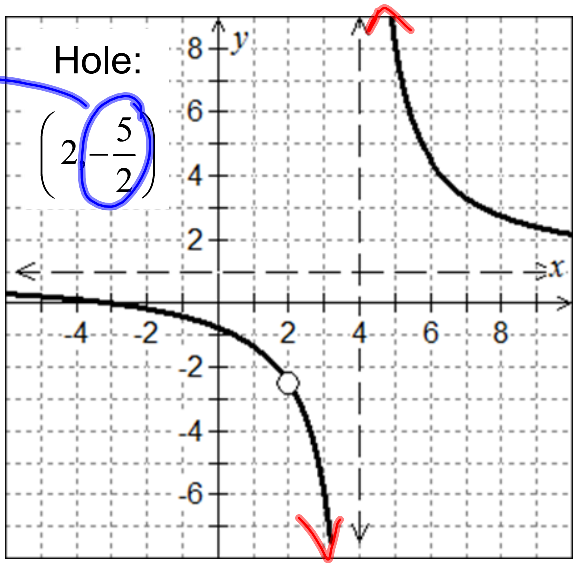
A.	B.	C.
$\lim\limits_{x\to 2}\frac{x^2+x-6}{x^2-6x+8}$	$\lim\limits_{x\to 4^+}\frac{x^2+x-6}{x^2-6x+8}$	$\lim\limits_{x\to 4^-}\frac{x^2+x-6}{x^2-6x+8}$
$\frac{2^2+2-6}{2^2-6(2)+8}=\frac{0}{0}$	$\frac{4^2+4-6}{4^2-6(4)+8}=\frac{14}{0}$	$\frac{14}{0}$
Indeterminant	Undefined	
point discontinuity (hole)	(Possible Vertical Asymptote)	

Graphically, you can see the limits of the function shown at right. Just because a function is undefined at a value of *x* doesn't mean that you can't find the limit. Use the graph of the function to determine the value of each limit below.

$$\lim_{x\to 2}\frac{x^2+x-6}{x^2-6x+8} = \underline{-\frac{5}{2}}$$

$$\lim_{x\to 4^+}\frac{x^2+x-6}{x^2-6x+8} = \underline{\infty}$$

$$\lim_{x\to 4^-}\frac{x^2+x-6}{x^2-6x+8} = \underline{-\infty}$$



What is the process for finding discontinuities of a rational function from pre-calculus?

1. Factor both numerator + denominator and simplify

2. A point discontinuity (hole) occurs if the factor that made the denominator=0 gets cancelled

3. A non removable discontinuity (asymptote) occurs when the factor that made denominator=0 <u>DOES NOT</u> get cancelled.

You can perform the same algebraic analysis to find the limit of the removable, or point discontinuities and the non-removable, or infinite discontinuities using what we will call the **Factoring or Cancellation Technique**.

$$\lim_{x\to 2}\frac{x^2+x-6}{x^2-6x+8} \Rightarrow \frac{(x+3)\cancel{(x-2)}}{(x-4)\cancel{(x-2)}} \Rightarrow \frac{x+3}{x-4}$$

Now plug in $x=2$

$$\frac{2^2+2-6}{2^2-6(2)+8}=\frac{0}{0}$$

asymptote @ $x=4$

hole @ $x=2$

$$\frac{2+3}{2-4}=\frac{5}{-2}$$

$$\lim_{x\to 2}\frac{x+3}{x-4}=-\frac{5}{2}$$

Notice the simplified expression above and consider the behavior of this function. Graphically there is a non-removable discontinuity commonly called a

Vertical asymptote

at $x = 4$. Because the *y*-values do not approach one specific value from both sides then the limit does not exist. By using the numerical, graphical, and algebraic techniques together, you can determine the behavior of the simplified function on either side of the vertical asymptote. This is true because the original function and the simplified function agree everywhere except at the

Point discontinuity (hole)

$(2, -\frac{5}{2})$.

Determining Behavior of a Function Using One-Sided Limits

$\lim_{x\to 4^+} \frac{x^2+x-6}{x^2-6x+8}$	$\lim_{x\to 4^-} \frac{x^2+x-6}{x^2-6x+8}$
As $x \to 4^+$	As $x \to 4^-$ smaller than 4
Pick a value: 4.1	Pick a value: 3.9
Simplified function: $\frac{x+3}{x-4}$	Simplified function: $\frac{x+3}{x-4}$
Plug in 4.1 $\frac{4.1+3}{4.1-4} = \frac{7.1}{.1}$ $\frac{(+)}{(+)} = \infty$	$\frac{3.9+3}{3.9-4} = \frac{6.9}{-.1}$ $\frac{(+)}{(-)} = -\infty$
$\lim_{x\to 4^+} f(x) = \infty$	$\lim_{x\to 4^-} f(x) = -\infty$

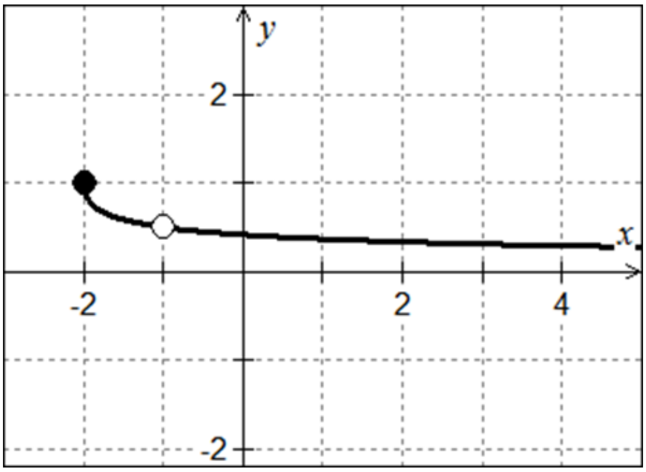
$\lim_{x\to 4} f(x) = \text{DNE}$ (left and right are NOT the same)

EX #3: The Rationalization Technique or Conjugate Method

A. The graph of $g(x)=\dfrac{\sqrt{x+2}-1}{x+1}$ is shown below.

The technique of rationalization can be used to find the limit. conjugate

$$\lim_{x\to -1}\frac{\sqrt{x+2}-1}{x+1}\cdot\frac{\sqrt{x+2}+1}{\sqrt{x+2}+1}$$



$$\frac{x+2-1}{(x+1)(\sqrt{x+2}+1)}=\frac{\cancel{x+1}}{\cancel{(x+1)}(\sqrt{x+2}+1)}$$

$$\lim_{x\to -1}\frac{1}{\sqrt{x+2}+1}\Rightarrow\frac{1}{\sqrt{-1+2}+1}=\frac{1}{2}$$

B. $\lim_{x \to 5} \dfrac{x-5}{3 - \sqrt{x+4}}$

EX #4: Find each of the following limits analytically. Show your algebraic steps.

A. $\lim_{x \to -2} x^3 + 3x^2 - 4x + 5$

B. $\lim_{x \to \frac{3}{2}} 2x^2(2x+3)$

C. $\lim_{x \to 3} (5x+1)^{\frac{2}{3}}$

D. $\lim_{x \to -4} \frac{2x^2+7x-4}{x^2-x-20}$

E. $\lim_{x \to 3} \frac{\sqrt{x+1}-2}{x-3}$

F. $\lim_{x \to 0} \frac{\sqrt{x+4}-2}{x}$

G. $\cancel{\lim_{x \to 0} \frac{\frac{1}{3} - \frac{1}{x+3}}{x}}$

$$\lim_{x \to 0} \frac{\frac{1}{\cancel{3}}^{3(x+3)} - \frac{1}{\cancel{x+3}}^{3(x+3)}}{\frac{x}{1}^{3(x+3)}}$$

LCD $3(x+3)$

$$\lim_{x \to 0} \frac{x+3-3}{3x(x+3)} \Rightarrow \frac{\cancel{x}}{3\cancel{x}(x+3)} \Rightarrow \frac{1}{3(x+3)}$$

$$\lim_{x \to 0} = \frac{1}{3(0+3)} = \frac{1}{9}$$

H. $\lim_{x \to 2^+} \frac{3x^2 - 7x + 2}{x^2 - 4}$

J. $\lim_{x \to 5^+} \frac{4x+3}{x-5}$

K. $\lim_{x \to 5^-} \frac{4x+3}{x-5}$